ST JOHNS SCHOOL

Mathematics B

Revision

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The aim of this document is to provide revision resources, equations, examples, etc. to primarily the original author, Scott Williams. With permission, this document may be used and/or edited by another person/s to extend the available knowledge provided in it.

1.0 Revising basic algebra – Rearranging linear equations

Rearranging equations is a necessity in high level maths, and is expected to be known in all areas. Below is a table to review on how it works and why it works.

Table 1 – *Rearranging Examples*

Starting Equation	**Step to isolate**	**Final equation**	**Why it works**
$A = L + P$	$A - L = L + P - L$	$P = A - L$	$L + -L = 0$
$v = dt$	$\frac{v}{d} = \frac{dt}{d}$	$t = \frac{v}{d}$	$\frac{dt}{d} = t$
$A = lw$	$\frac{A}{w} = \frac{lw}{w}$	$l = \frac{A}{w}$	$\frac{lw}{w} = l$
$F = \frac{kQq}{r^2}$	$\sqrt{\frac{kQq}{\frac{kQq}{r^2}}} = \sqrt{\frac{kQq}{F}}$	$r = \sqrt{\frac{kQq}{F}}$	$\frac{kQq}{\frac{kQq}{r^2}} = r^2$ And $\sqrt{r^2} = r$

Table 2 – *Rules to Remember*

Start	End	Why it works
$x = y + z$	$y = x - z$	Because $z - z = 0$
$x = y - z$	$y = z - x$	Because $-z + z = 0$
$x = yz$	$y = \frac{x}{z}$	Because $\frac{z}{z} = 1$
$x = \frac{y}{z}$	$y = xz$	Because $z \cdot \frac{1}{z} = 1$
$x = \frac{z}{y}$	$y = \frac{z}{x}$	Because I just showed it above!
$x = y^z$	$y = \sqrt[z]{x}$	Because $\sqrt[z]{y^z} = y$
$x = \sqrt[z]{y}$	$y = x^z$	Because $\left(\sqrt[z]{y}\right)^z = y$

2.0 Finding *x* and *y* intercept

To find *x* and *y*, let the other equal 0 in the equation $y = mx + c$

Table 3 – *Finding x and y Examples*

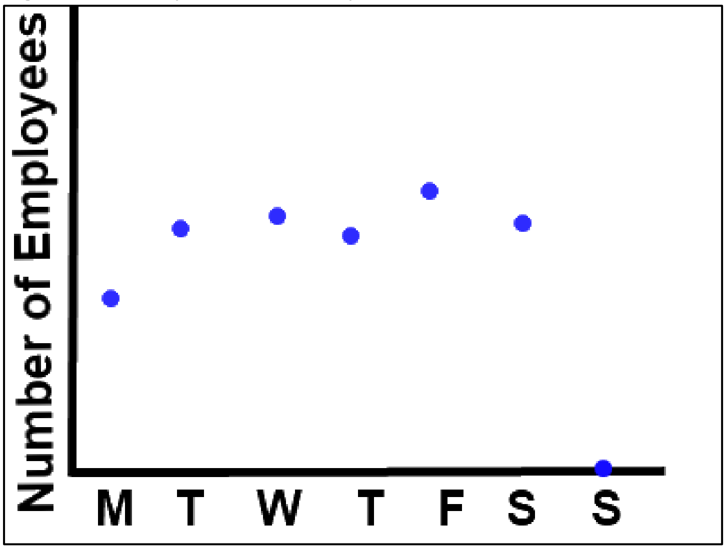
Example 1	Example 2	Example 3	Notes
$y = 6x + 18$	$y = 5x + 12$	$y = 3x - 21$	Equation
$y = 0 + 18$	$y = 0 + 12$	$y = 0 - 21$	Finding *y* intercept
$0 = 6x + 18$	$0 = 5x + 12$	$0 = 3x - 21$	Finding *x* intercept
$-18 = 6x$	$-12 = 5x$	$21 = 3x$	
$x = -3$	$2.4 = x$	$7 = x$	

In the above table (Table 3), we are finding the *x* and *y* intercept in their respected equations. Even though the *y*-intercept is visible in the equation $y = mx + c$ as *c*, the above proves that this is correct.

3.0 Independent VS Dependant – Continuous VS Discrete

Golden Rule 3: ***The Independent Axis is x, the Dependent Axis is y.***

Figure 1 – *Independent VS Dependent*



As Figure 1 suggests, the number of employees attending work **depends** on the day of the week.

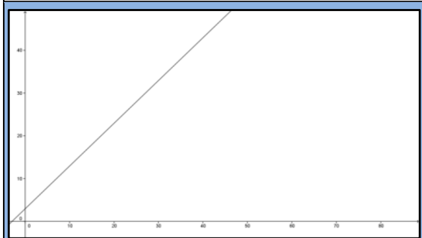
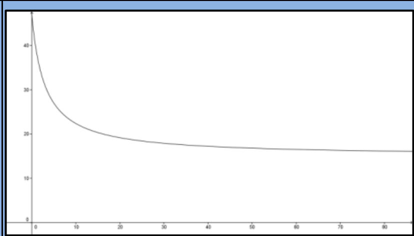
You can count discrete data, but you can only measure continuous data.

Discrete data is usually dependent, and is only certain points. For example, the data in Figure 1 or Figure 2 bellow. Some examples of discrete data include:

- *Money*
- *Objects*
- *People*

Continuous data is usually independent, and is every single integer. For example, the Figure 2 shown below. Some examples of continuous data include:

- *Temperature*
- *Time*
- *Date/Days*

Figure 2 – *Discrete Data*	**Figure 3 – *Continuous Data***
	
Domain: $x \in [\mathbf{10, 15, 20, 25, 30} \ldots]$ **Range:** $y \in [3,6,9,12 \ldots]$	**Domain**: $x \in \mathbb{R}^+$ **Range:** $y \in (15,43]$

4.0 Domain and Range

Within any graph, the **domain** is how low and high the *x* value may be. Once the equation becomes invalid (Example: $\boldsymbol{f(-1) = \sqrt{-1} + 4}$) then the domain has reached its maximum potential. Also within a graph is the **range**. This is how low and high the *y* value may be, similar to the domain.

Below are the three ways to represent domain and range. When displayed, **[**, ● and $\leq$ shows that the value is included, and **(**, ○ and $\leq$ shows that it is not included.

Remember, ***All graphs, be functions or relations, have a domain and range***

Table 4 – *Domain and Range examples*

Line	Brackets	Equation	Element
●————————●	$[6,12]$	$6 \leq x \leq 12$	$x \in [6,8,10,12]$
●————————○	$[6,12)$	$6 \leq x < 12$	$x \in (6,8,10,12]$
○————————○	$(6,12)$	$6 < x < 12$	$x \in (6,8,10,12)$
6 8 10 12			

5.0 Hyperbolas

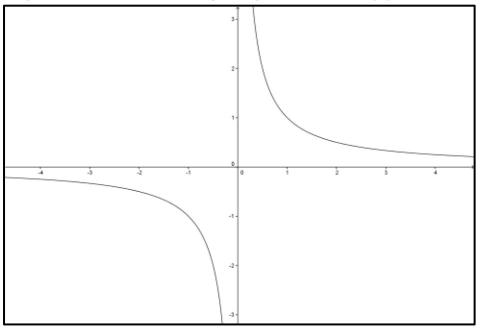
A hyperbola is a special type of graph, similar to x^2. It is a continuous graph which never, ever touches zero. The default equation for a hyperbola is $f(x) = \frac{1}{x}$ which explains why the graph never touches zero. No matter what number you divide by one, it will never equal zero. Table 5 below proves this. Figure 4 is also an image of a basic asymptote with no adjustments made to the function.

Remember, ***Hyperbolas will never touch their origin, which is not always equal to zero.***

Table 5 – *Example values for a base hyperbola*

$x =$	10	10000	1000000	100000000	10^{40}
$f(x) =$	0.1	0.0001	0.000001	0.00000001	10^{-40}

Figure 4 – *An image of a base hyperbola*



Domain: $x \in \mathbb{R}\backslash[\mathbf{0}]$
Range: $y \in \mathbb{R}\backslash[\mathbf{0}]$

6.0 Functions VS Relations

Even though we often associate graphs with functions, **relations** are also possible. One often used example of a relation is a circle shape, as shown in Figure 5. The easiest and most simple way to tell a function apart from a relation is called the vertical line test. If the graph is a function, the line will only touch it once at some point or not at all. If the graph is a relation, the line will either not touch it or cross through twice.

Figure 5 – *Vertical Line Test on a relation*

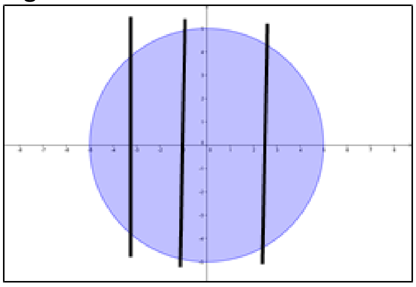
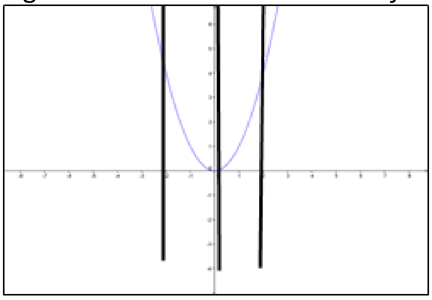


Figure 6 – *Vertical Line Test on a function*



7.0 Hybrid Functions

Hybrid functions are those that consist of different rules or equations for different subsets or parts of the domain. For example:

$$f(x) = \begin{Bmatrix} x+1, & \text{for } x \leq 0 \\ x^2, & \text{for } x > 0 \end{Bmatrix}$$

The above is a hybrid function which obeys the rules $y = x + 1$ if $x \leq 0$ and $y = x^2$ if $x > 0$. Figure 7 below is an example of how to display hybrid functions.

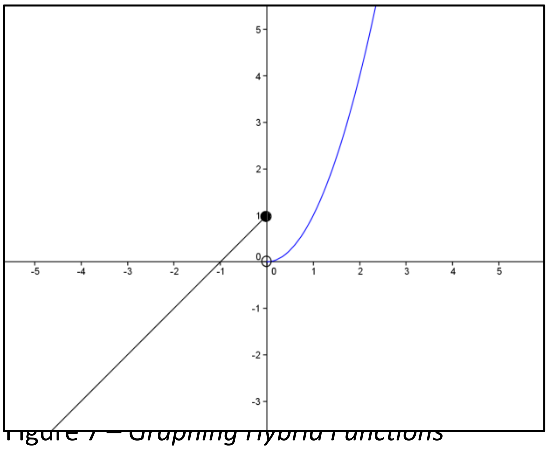


Figure 7 – *Graphing Hybrid Functions*

Remember, ● means the value is included in the domain and/or range, and **O** means the value is excluded.

Table 6 – *Domain and Range of a Hybrid Function*

DAR of First Function	DAR of Second Function	DAR of Entire Function
$x \in (-\infty, 0]$	$x \in (0, \infty)$	$x \in R$
$y \in (-\infty, 1]$	$y \in (0, \infty)$	$y \in R$

8.0 Equation of a Line

When we are given any one point and a slope, or two points and no slope, we can find the equation of that particular function or relation. This can be done using these two methods:

$y - y_1 = m(x - x_1)$ where $\boldsymbol{x_1, y_1}$ is co-ordinate on the graph and $\boldsymbol{m}$ = the slope

$y - y_1 = (\frac{y_2 - y_1}{x_2 - x_1})(x - x_1)$ where $(\boldsymbol{x_1, y_1})$ and $(\boldsymbol{x_2, y_2})$ are any two co-ordinates on the graph

Example 1:

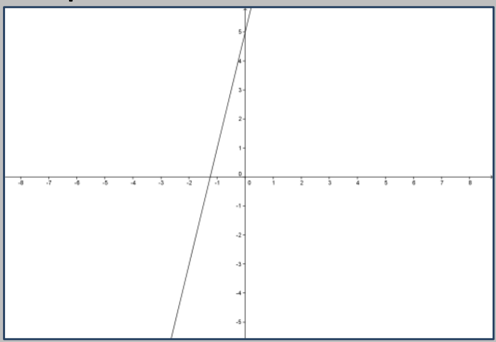


Figure 8 – *A graph with the equation* $y = 4x + 5$

Figure 8 has a slope of 4, and has a co-ordinate of (-1.25,0).

$y - y_1 = m(x - x_1)$
$y - 0 = 4(x + 1.25)$
$y = 4x + 5$
∴ the graph on Figure 8 has the equation $y = 4x + 5$, which is **true.**

Example 2:

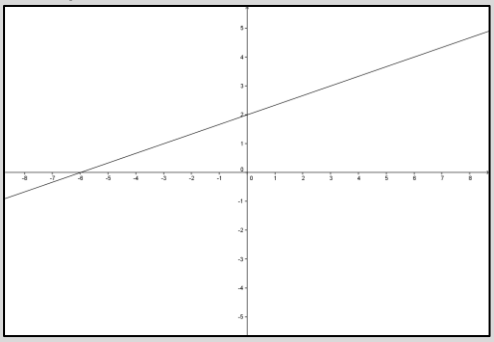


Figure 9 – *A graph with the equation* $y = \frac{1}{3}x + 2$

Figure 9 slope is unknown, and has the co-ordinates of (-3,1) and (4.08,3.36).

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 1 = \left(\frac{3.36 - 1}{4.08 + 3}\right)(x + 3)$$

$$y - 1 = \frac{1}{3}(x + 3)$$

$$y = \frac{1}{3}x + 2$$

∴ the graph on Figure 9 has the equation $y = \frac{1}{3}x + 2$, which is **true.**

9.0 Linear Modelling

Using functions is majorly and most often applied to real life situation. It can be used in many situations which involve time, money, or any independent VS dependent situation.
Example: *A taxi company charges $5 upfront, then an extra $6 per kilometer.*

$\boldsymbol{y = mx + c}$
y = Cost Per Unit · x + Fixed Cost
$\boldsymbol{y = 6x + 5}$

∴ the price of a service from this particular taxi company can be expressed in the function:
$f(x) = 6x + 5$

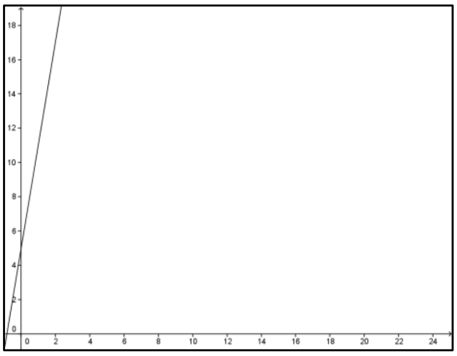


Figure 10 - $f(x) = 6x + 5$, the function of a taxi company

10.0 Circles

Circles are a particular type of relation expressed with the equation $x^2 + y^2 = r^2$, where $r =$ the radius. A vertical line test proves that a circle is not a function, as shown in Figure X.

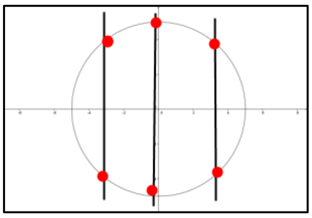


Figure 11 – *A circle relation, with a vertical line test. It has a radius of 5 and the center point (0,0).*

A circle can also be expressed with the equation $(x - h)^2 + (y - k)^2 = r^2$ in which h and k are the center of the circle's x and y coordinates. The circle in Figure 11 has the center point of 0, because h and k values are both 0. But if we used the equation $(x - 3)^2 + (y - 2)^2 = 5^2$, this would hereby move the center of the circle 3 units to the right on the x-axis, and 2 units up on the y-axis. Figure 12 (below) proves this.

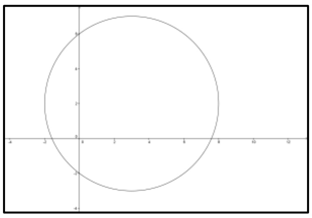
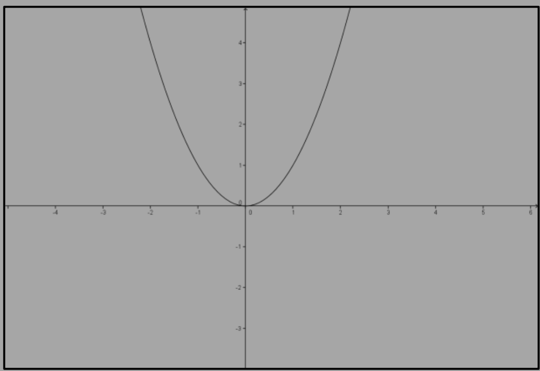
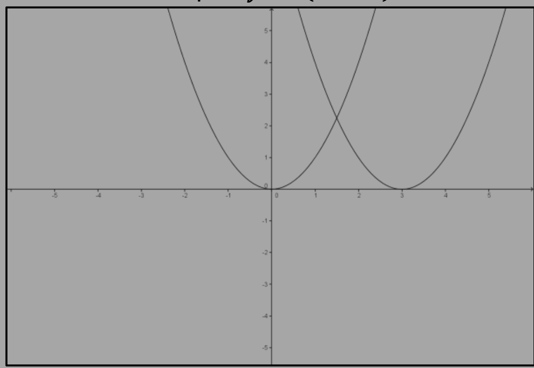
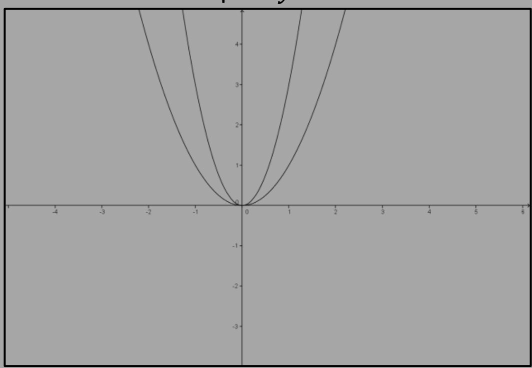
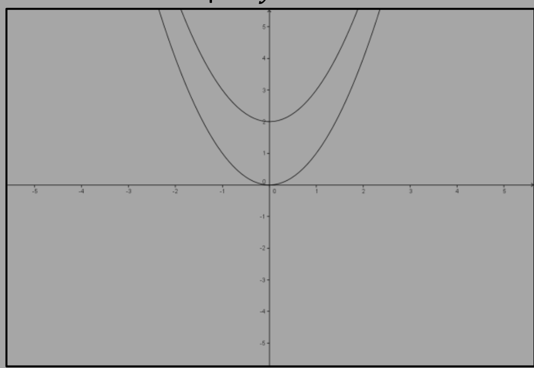
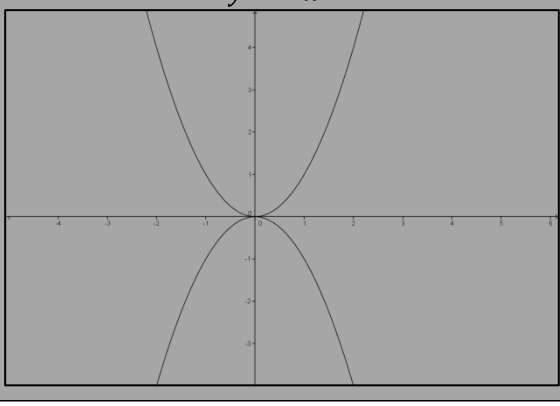


Figure 12 – *The same relation as Figure 11, but with its center point moved 3 to the right and 2 up.*

11 Transforming Functions

Below are all the different and various methods to transform functions, and the effect they each have on the graph. The example used is $f(x) = x^2$, but all these methods work for all functions, and most relations.

Table 7 – *Modifying Equations*

Turning point $y = x^2$ 	**Horizontal translation** $y = (x - h)^2$ Example: $y = (x - 3)^2$ 
Dilation about the *y* axis $y = ax^2$ Example: $y = 3x^2$ 	**Vertical translation** $y = x^2 + k$ Example: $y = x^2 + 2$ 
Reflection in *x* axis $y = -x^2$ 	**Wave** (not on exam) $y = \cos(x)$ $y = \tan(x)$ $y = \sin(x)$